

Automated Mitosis Detection in Color and Multispectral High-Content Images in Histopathology: Application to Breast Cancer Grading in Digital Pathology

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Context

- Digital Pathology
- Breast Cancer Grading
- Research Contributions
 - Mitosis Detection Framework for Color Images
 - Mitosis Detection Framework for Multispectral Images
 - Dynamic Sampling Framework for WSI analysis
- Conclusion & Future works

Ipal Image & Pervasive Access Lab

Digital Pathology

Slide Preparation

- Dissection / extraction
- Chemical processing
- Sectioning
- Staining Hematoxylin & Eosin (H&E), Immunohistochemistry (IHC)

Imaging & Visualization

- Microscope
- Slide Scanner



Biopsy glass slide



Scan at 40X Resolution: 0.2456µm per pixel





Pathology vs Digital Pathology

Pathology

- Manual analysis is labor intensive work
- Inter and intra-reader variations

Digital Pathology

- Location independence
- Virtual slide sharing with clinicians
- Enables automated analysis techniques





Breast Cancer Grading

Breast Cancer

- Origin Ductal, lobule or stromal tissues
- Proliferation Carcinoma in Situ, Invasive
- Worldwide, Breast Cancer accounts for 22.9% of all cancer in women [1]
- I in 8 US women estimated to develop Invasive Breast Cancer over the course of her lifetime [2]

➢ Nottingham Grading System for Invasive Breast Cancer

- International grading system recommended by the World Health Organization
- 3 Criteria [3]:
 - I. Gland Formation
 - 2. Nuclear Atypia / Pleomorphism
 - 3. Mitosis Count
- I) Peter Boyle, et al. World cancer report 2008. IARC Press, International Agency for Research on Cancer, 2008.
- 2) US Breast Cancer Statistics, 2013 (<u>http://www.breastcancer.org/symptoms/understand_bc/statistics</u>).
- 3) CW Elston & I O Ellis, Pathological prognostic factors in breast cancer. Histopathology, 19(5):403-410, 1991.



Mitosis Count

- Scan sections to find area with most mitotic activity (often at tumor edge)
- ➢ Count mitosis in 10 consecutive high power fields (HPFs) of selected area
- ➢ Skip fields with few carcinoma cells or obvious necrosis
- ➢ Score I (< 10 Mitosis)</p>
- ➢ Score 2 (10 ~ 19 Mitosis)
- Score 3 (> 19 Mitosis)

HPF is an area of microscope field diameter of 58mm (or a digitized square image 512 * 512 μm²).



- I. State-of-the-art in nuclei detection, segmentation and classification
 2. Color Framework
 - Selection of color channels for different tissue component
 - Intensity (Ist order statistical) and texture (2nd order statistical) features
 - Region vs patch based texture features analysis for mitosis discrimination
 - An inspection of over-sampling method for balancing the training set
- 3. Multispectral Framework
 - Spectral absorption responses of different tissue components
 - Multispectral-statistical features in selected Spectral Bands (SBs)
- 4. Whole Slide Image Analysis Framework
 - Robust strategy to explore WSI using a dynamic sampling framework
 - Extension of ITK QuadEdgeMesh data structure to handle duality of meshes
- 5. Proof of concept in MICO platform



MITOS Benchmark (ICPR 2012)

- ➤ 5 breast cancer biopsy slides (H&E stained) provided by [4]
- ➤ In each biopsy slide, I0 HPFs at 40X magnification are selected
- ➤ 35 Training HPFs 226 mitotic nuclei (69.3%)
- ► I5 Evaluation HPFs I00 mitotic nuclei (30.7%)

Scanners	Resolution per pixel	HPF dimension to cover area of $512 \times 512 \ \mu \text{ m}^2$		
Aperio Scanner	0.2456 µm	2084 ×2084 pixels		
Hamamatsu Scanner	0.2273 × 0.22753 µm	2252 ×2250 pixels		
Multispectral Microscope	0.185 µm	2767 × 2767 pixels		

4) Team of Prof. Frederique Capron, head of Pathology department, Pitie-Salpetriere Hospital Paris.



Mitosis Detection – A Challenging Problem

Mitosis Detection – A Challenge

- > Mitotic nuclei have
 - large variety of shape configuration
 - Texture variation
 - Low frequency of appearance in HPF
 - Similarity with other types of objects
 - (e.g., apoptosis, necrosis, dust particles etc.)



Dust particles



Apoptosis



Mitosis



Mitosis Detection – A Challenge

> Which one is mitotic nuclei and which is not?





Mitosis Detection – A Challenge

> Which one is mitotic nuclei and which is not?





Intensity, Texture & Morphology based Mitosis detection in Color images (ITM²C) Framework



ITM²C Framework



Histogram of Selected Channels on Aperio Dataset





Histogram of Selected Channels on Hamamatsu Dataset







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ITM²C Framework Step 3

- Morphology Features (5 Features)
 - Area, Perimeter, Roundness, Elongation, Equivalent spherical perimeter
- Intensity Features (5 Features)
 - ► Mean, Median, Standard Deviation, Skewness, Kurtosis
- Texture Features (18 Features)
 - ➢ Co-occurrence Features (8 Features)
 - Correction, cluster shade, cluster prominence, energy, entropy, Hara-correlation, inertia, difference moment
 - ➢ Run-Length Features (10 Features)
 - SRE, LRE, GLN, RLN, LGRE, HGRE, SRLGE, SRHGE, LRLGE, LRHGE
- Compute intensity and texture features for each color channel (total eight channels)
- Total Features = $5 + \frac{8}{17}(5 * 18) = 189$



Threshold & Morphology Segmentation & Candidate Selection • Feature Computation Feature Normalization & Selection ٠ Handling Imbalance Training set Classification

ITM²C Framework Step 4

Feature Normalization

 $f\uparrow' = f - f\downarrow min \ / f\downarrow max - f\downarrow min$

where ff' is normalized feature, f is actual feature

• Feature Selection using Consistency subset evaluation method

$Consistency \downarrow s = 1 - \sum_{j=0}^{j=0} \int J = \frac{D \downarrow_j}{|D \downarrow_j|} - \frac{M \downarrow_j}{|N|}$

where *s* is a feature subset, *j* is the number of distinct combination of features for s, |Dlj| is the number of occurrences of the *j*th feature combination, |Mlj| is the cardinality of the majority class for the *j*th feature combination and *N* is the total number of instances

Use these subsets in conjunction with a hill climbing search method, augmented with backtracking





ITM²C Framework Step 5

Handling Imbalanced Training Set

- High degree of imbalance in training set (mitosis vs non-mitosis)
- Down-sampling of non-mitosis
- Over-sampling of mitosis using Synthetic Minority Over-sampling TEchnique (SMOTE)
 - 2 neighbors are selected from 5-nearest neighbors
 - New instance is generated in the direction of selected 2 neighbors











ITM²C Classification Results using Single Channel Features



On Aperio Dataset

On Hamamatsu Dataset



ITM²C Result using All vs Selected Features on Evaluation set

		Aperio Dataset			Hamamatsu Dataset		
Features	Classifiers	TPR	PPV	FM	TPR	PPV	FM
All Features	DT	65%	71%	67.71%	60%	62%	60.91%
	MLP	68%	69%	68.34%	60%	61%	60.61%
	LSVM	72%	66%	68.57%	61%	62%	61.31%
	NLSVM	58%	83%	68.24%	53%	73%	61.27%
Selected Features	DT	67%	73%	69.79%	61%	64%	62.56%
	MLP	66%	74%	69.84%	60%	66%	62.83%
	LSVM	74%	71%	<u>72.55%</u>	63%	66%	<u>64.62%</u>
	NLSVM	59%	84%	69.41%	55%	74%	63.22%

TPR = True Positive Rate PPV = Predictive Positive Value FM = F-Measure



Different Patch Sizes for Feature Computation

Patch Size in pixels	Aperio Dataset (µm)	Hamamatsu Dataset (µm)		
Patch 80 × 80	19.648 × 19.648	18.184 × 18.202		
Patch 70 × 70	17.192×17.192	15.911 × 15.927		
Patch 60 × 60	14.736 × 14.736	13.638 × 13.652		
Patch 50 × 50	12.28 × 12.28	11.365 × 11.377		
Patch 40 × 40	9.824 × 9.824	9.092 × 9.101		



Mitosis Patch from Aperio Dataset

Mitosis Patch from Hamamatsu Dataset



Region vs Patch Features based Classification using LSVM Classifier

• Both scanners have different information on same patch size.



On Aperio Dataset

On Hamamatsu Dataset



Comparison of Results with ICPR MITOS Contest 2012





Candidate Classification on Aperio Dataset (TP=Green, FP=Yellow, FN-Blue)





Mitosis Detection in Multispectral Images

Multispectral Dataset

- 10 Spectral bands
- 17 Focal Planes (layer Z-stack)
- 4 images per HPF
- | HPF = 4 * |7 * |0 = 680 images
- Image resolution = $251.6 \times 251.6 \mu m^{12}$ an area of $0.063 mm^{12}$
- Total 50 HPF (322 mitosis)
 - 35 Training set (244 mitosis)
 - 15 Evaluation set (98 mitosis)



Spectral bands (SBs) of multispectral microscope and example of each SB



Multispectral Intensity, Texture & Morphology-based Mitosis detection in Multispectral images (MITM³) Framework



MITM³ Framework



Mitosis Detection in Multispectral Images

Spectral Band Selection

Method I – Tissue Spectral Absorption



Multispectral Image (SB 8)

Color (RGB) Image



Normalized average gradient spectra of four tissue components



Spectral Band Selection

Method 2 – H & E Spectral Absorption



Normalized plot of the Hematoxylin (Blue line) and Eosin (Red line) dye absorption spectra and difference of Hematoxylin and Eosin (green line)



Spectral Band Selection

Method 3 – SBs Selection using Minimum redundancy and Maximum Relevance (mRMR)

- Relevance $D=1/|S| \sum s \downarrow i \in S^{\uparrow} MI(s \downarrow i; c \downarrow j)$
- Redundancy $R=1/|S|/2 \sum s \downarrow i, s \downarrow j \in S^{\uparrow} MI(s \downarrow i; s \downarrow j)$
- Mutual Information

 $MI(S;C) = -\sum s \downarrow i \in S \uparrow = p(s \downarrow i) \log \downarrow 2 \ (p(s \downarrow i)) + \sum s \downarrow i \in S \uparrow = \sum c \downarrow j \in C \uparrow = p(s \downarrow i, c \downarrow j) \log \downarrow 2 \ (ps \downarrow i c \downarrow j)$

• Incremental search method is used to find the *n* SBs from the set $\{S\downarrow T - S\downarrow n-1\}$ by maximizing $\max_{\tau} s\downarrow i \in S\downarrow T - S\downarrow (n-1) [MI(s\downarrow i; c) - 1/n - 1 \sum s\downarrow j \in S\downarrow (n-1) \uparrow MI(s\downarrow i; s\downarrow j)]$

S = SBs set, C = class label, $p(s\downarrow i)$ = probability density function of SB $s\downarrow i$, $ps\downarrow i c\downarrow j$ = conditional probability density function of SB $s\downarrow i$ and class label $c\downarrow j$



Spectral Band Selection

Method 3 – SBs Selection using Minimum redundancy and Maximum Relevance (mRMR)

SBs	MI	Accumulated MI	Accumulated MI%
SB 8	3.60	3.60	33%
SB 9	3.59	0.95	42%
SB 7	3.58	0.94	51%
SB 6	3.18	0.93	60%
SB 2	3.16	0.92	69%
SB I	3.11	0.91	78%
SB 3	3.05	0.89	86%
SB 0	2.99	0.88	91%
SB 4	2.94	0.85	95%
SB 5	2.85	0.82	100%



3 Rankings of Spectral Bands

	Method I – Tissue Absorption Spectra		Method 2 – H&E Absorption Spectral		Method 3 – mRMR		
SBs	Mitosis-Cytoplasm	SBs	H-E	SBs	MI		
SB 7	0.47	SB 7	0.96	SB 8	3.60		
SB 8	0.45	SB 8	0.91	SB 9	3.59		
SB 9	0.36	SB 9	0.64	SB 7	3.58		
SB 3	0.33	SB I	0.39 SB 6		3.18		
SB 2	0.31	SB 6	0.33	SB 2 3.16			
SB 6	0.30	SB 0	0.23	SB I	3.11		
SB I	0.30	SB 2	0.23	SB 3	3.05		
SB 4	0.29	SB 3	3 0.21 SB 0		2.99		
SB 0	0.28	SB 5	0.04	SB 4	2.94		
SB 5	0.27	SB 4	0	SB 5 2.85			

• Selected SBs are 8,9,7,6,2,1,3 and 0.



Region vs Patch Features based Classification Results



Region vs Patch features based classification results using LSVM classifiers



MITM³ Classification on Evaluation Set

		All SBs Features			Selected 8 SBs Features		
Features	Classifiers	TPR	PPV	FM	TPR	PPV	FM
	DT	67%	53%	59.1%	62%	62%	62.24%
Region	MLP	64%	56%	59.72%	62%	66%	63.10%
Features	LSVM	63%	60%	61.69%	64%	62%	63.32%
	NLSVM	54%	68%	60.23%	59%	69%	63.74%
	DT	61%	71%	65.57%	65%	70%	67.72%
Patch size	MLP	63%	67%	64.92%	66%	70%	68.06%
I 6.65 μm Features	LSVM	69%	75%	<u>71.96%</u>	74%	73%	<u>73.74%</u>
	NLSVM	55%	77%	64.29%	59%	77%	67.05


Plot of TPR, PPV and FM using Single SB Features with LSVM classifier





Plot of FM using Accumulated Features from the order of mRMR Selection





Results on different subsets of Features using 5-Fold Cross Validation



MorF = Morphology Features MSIF = Multi-Spectral Intensity Features MSTF = Multi-Spectral Texture Features MSITF = Multi-Spectral Intensity & Texture Features MMSF = Morphological & Multispectral Statistical Features



Comparison of MITM³ Framework with ICPR 2012 MITOS Contest





Comparison of ITM²C and MITM³ Frameworks





Whole Slide Image (WSI) Analysis

Switching from HPF to WSI Analysis





Orientable 2-Manifold Meshes and Existing Data Structure

- *itk::QuadEdgeMesh* existing data structure in ITK can handle discrete 2-manifold surfaces
- A constant complexity local access on modifications





Orientable 2-Manifold Meshes and NewData Structure

- Proposed an extension of existing ITK data structure for Orientable 2-manifold meshes to handle duality
- *itk::QuadEdgeMeshWithDual* new data structure and a filter that transform primal mesh to primal/ dual mesh

		Old Structure	New Structure	
Changes	OriginRefType	Point ID, Cell ID	Pair< Point, Cell >, Pair< Cell, Point >	
Additions	Dual Containers	_	Dual Pointers, Cells and EdgeCells Containers	





Planer Delaunay/Voronoi Mesh and Non-Planer Triangulation/Simplex Mesh





MICO Project (ANR TecSan)

- COgnitive virtual MIcroscope for Breast Cancer Grading (MICO) Project
 - Funded by French National Research Agency (ANR)
 - Launched in Feb, 2011 Jul, 2014 (3.5 Years)







MICO 2.0 Architecture









Dynamic Sampling applied over WSI: Incrementally Voronoi Diagram



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Stereology Framework for evaluation of ITM²C framework in MICO

- > TerritoryExtractor
- ➢ FrameGenerator
- ➢ FrameSampler 3×3
- FrameSampler 4×4
- ➢ ITM2C Framework
- MitosisScorer



Stereology Flow used for Mitosis Score over a ROI



WSI analyzed by ITM²C Framework are displayed on Calopix platform

The color code is based on the number of mitosis detected in the frame (from blue for zero mitosis to red for 10 or more mitosis.





Mitosis Detector Integration in Calopix





Conclusion & Future works

Summary

- Proposed automated mitosis detection framework for different scanners and multispectral microscope
- Efficient and generic strategies (Stereology & Dynamic Sampling) to explore WSI
- Evaluation of these frameworks in MICO platform

Future work

- Expand proposed frameworks from two-class problem to multi-class problem and classify other microscopic objects like lymphocytes, apoptosis, normal nuclei, cancer nuclei
- Main area of Interests:
 - Machine Learning
 - Computer Vision
 - Pattern Recognition
 - Medical Image Analysis



Conclusion & Future works

Summary

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Intellectual Property

"MitosisDetector – Mitosis Detector for Histopathology", H. Irshad, L. Roux, D. Racoceanu, Copyright CNRS (CNRS Statement Software) No. DL 05963-01 for 2955 IPAL UMI, 2013.

Journals

- H. Irshad, A. Gouaillard, L. Roux, D. Racoceanu, "Multispectral Band Selection and Spatial Characterization: Application to Mitosis Detection in Breast Cancer Histopathology", in *Computerized Medical Imaging and Graphics* (CMIG), (Submitted).
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- 8. H. Irshad, A. Gouaillard, L. Roux, D. Racoceanu, "Spectral Band Selection for Mitosis Detection in Histopathology", in *11th* International Symposium on Biomedical Imaging (ISBI), Beijing China, 2014.
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Mitosis Detection in Color Images

Receiver Operating Characteristic (ROC) curve of patch based features with LSVM Classifier



On Aperio Dataset

On Hamamatsu Dataset



Mitosis Detection in Color Images

Candidate Classification on Hamamatsu Dataset TP=Green, FP=Yellow, FN-Blue)





Classification Results with White, Red, Green and Blue SBs using 5-Fold Cross Validation

Features	Classifiers	TPR	PPV	FM
	DT	51%	63%	56.11%
Red SBs	MLP	48%	71%	57.46%
(SB 0,8,9)	LSVM	67%	56%	61.19%
	NLSVM	49%	75%	59.19%
	DT	50%	68%	57.55%
Green SBs	MLP	50%	65%	56.84%
(SB 5,6,7)	LSVM	65%	58%	61.14%
	NLSVM	48%	78%	59.39%
	DT	43%	59%	49.82%
Blue SBs	MLP	46%	69%	55.49%
(SB 2,3,4)	LSVM	54%	60%	56.81%
	NLSVM	46%	75%	56.65%
	DT	42%	65%	51.32%
White SB	MLP	44%	74%	55.15%
	LSVM	56%	52%	54.11%
	NLSVM	44%	77%	55.84%





Dynamic Sampling for Cyto-Nuclear Atypia Score

- A dynamic sampling framework was developed based on computational geometry for Cyto-Nuclear Atypia (CAN) evaluation to avoid exhaustive analysis on WSI
- Main steps of method are:
 - I. Pathologest annotated territories by observing WSI using Calopix user interface
 - 2. Territories are extracted from WSI and split into several HPF frames
 - 3. 50 HPF are randomly selected for computation CNA scores using Christophe and Maria method [5]
 - 4. These scores are used for initialization of Voronoi diagram
 - 5. Next HPF is selected based on two criteria
 - I. At least one of its neighboring Voronoi cells has a high score that control the convergence of method towards areas with high score
 - 2. The distance between the new sample and its neighbors is not too short that prevents oversampling
 - 6. The final overall CNA score is the grade of the most atypia frame
- 5) Christophe & Maria, Marked point processes with simple and complex shape objects for cell nuclei extraction from breast cancer H&E images, SPIE Medical Imaging, 2013.



Dynamic Sampling Algorithm

Input: Current frames E, Voronoi Diagram VD_F, p, d, max_F **Output:** updated frames E, Voronoi Diagram VD_F, max_F Compute V_F Sort $V_{\rm F}$ according to decreasing distance to E for every $x \in VE$ do if Distance(x, E) > d then if $MaxScore(x) > p \times max_F$ then $\mathsf{E} = \mathsf{E} \cup \{x\}$ Update VD_F $\max_{F} = \max(S(x), \max_{F})$ break loop end if

Else

Break loop

End if

End for